Departure Time Distribution in the Stochastic Bottleneck Model

Hao Li[*1] Piet HL Bovy[*2] Michiel CJ Bliemer[*3]
Delft University of Technology [*1,2,3]
(Stevinweg 1, P.O. Box 5048, 2600 GA Delft, The Netherlands) [*1,2,3]
(0031 15 2784030, H.Li@tudelft.nl) [*1]
(0031 15 2784611, p.h.l.bovy@tudelft.nl) [*2]
(0031 15 2784874, m.c.j.bliemer@tudelft.nl) [*3]

This paper presents an analytical investigation of departure time choice under stochastic capacity using Vickrey's bottleneck model. The deterministic bottleneck model of Vickrey has been extended with random capacities. It is assumed that travelers are fully aware of the stochastic properties of the travel time and schedule delays at all departure times that emerge from day to day due to random capacity. Based on the analytical analyses, consideration of random capacities and travel time reliability in the utility function appears to result in significant shifts in the temporal demand pattern. Travelers will depart earlier and departure flows become more spread over a longer time period. Long term equilibrium is achieved under stochastic capacities. These theoretical findings are supported by empirical findings of delays at bottlenecks. Correctly modeling departure time distributions under stochastic supplies over days plays a crucial role in the assessment of the network performance and in the evaluation of dynamic traffic management measures.

Keywords: *Vickrey's bottleneck model, departure time choice, stochastic capacity, travel time reliability*

1. Introduction

It is well-known that waiting time losses at bottlenecks in transport networks cause travelers to adapt their travel timing such as to minimize their total trip cost from origin to destination. The basic model to study the impact of bottleneck capacity on travel time delays and the resulting departure time pattern is the well-known Vickrey's bottleneck model [1]. This model assumes a single bottleneck with constant capacity with a given total demand larger than capacity during a limited period, the usual peak, and known preferred arrival times (PAT) of the travelers. The model predicts the equilibrium temporal distribution of demand based on the behavioral assumption that travelers chose their departure times so as to minimize their individual travel costs that consists of waiting time and so-called schedule delay costs of arriving earlier or later than preferred. This bottleneck model has been analyzed by many others, see e.g. [2], [3], [4] and [5].

The purpose of this paper is to extend Vickrey's bottleneck model to the case of stochastic capacity of the bottleneck. We assume a certain random day-to-day variation in the capacity due to all kinds of causes such as weather volatility, incidents, traffic composition, and the like. In addition we assume that travelers using that bottleneck because of their long term experiences are fully informed about properties of the random capacity distribution. This allows us to derive mathematically the equilibrium temporal demand distribution (departure time pattern) that will emerge in the stochastic capacity case if a long term equilibrium exists.

In this paper we will elaborate on this stochastic bottleneck case and on the behavioral assumptions about the bottleneck users. We will present the resulting formulae and graphs from which the impact of stochastic day-to-day variations on the emerging temporal demand pattern can be deduced. It appears that consideration of random capacity leads to significantly different departure time patterns than in the deterministic case including a shift towards earlier departures.

2. Behavioral assumptions

We assume a bottleneck with a random fluctuation of capacity from day to day and a fixed number of travelers N using that bottleneck under congested condition (demand rate being larger than capacity), constant from day to day. Due to the capacity fluctuations, daily waiting times at the bottleneck are random as well, and so are the daily arrival times implying that daily schedule delays also are random. Due to his long term experience in using the bottleneck we assume that each of these travelers is aware of the stochastic properties of the waiting times and schedule delays at all departure times (i.e. expectations and variances) that emerge from day to day due to the random capacity, without being able to predict the daily traffic state before starting his commuter trip. Given this, we further assume that each traveler chooses an optimal departure time so as to minimize his long term future trip costs according to an individual cost function that includes cost components such as waiting time, departure schedule delays, arrival schedule delay, and reliabilities of these components. So, the traveler does not bother about traffic states at single days. That means we are looking for the long-term equilibrium pattern of departure times if it exists. For simplicity sake, in this paper we assume a fixed set of homogeneous travelers of size *N*, all having the same cost function and the same preferred arrival time.

3. Deterministic capacity case

The equilibrium for user departure time choice results when no traveler can reduce his travel cost by unilaterally altering his departure time [6]. In case of deterministic capacity, the travel time for departure time *t* is deterministic. Thus the travel cost for a user departing at a time instant *t* is simply composed of only two parts: travel time and schedule delays, formulated as (see [1]):

$$c(t)=\begin{cases} c_{free}+\alpha\cdot\tau(t)+\gamma_1\left(t^*-\left(t+\tau(t)\right)\right), \text{for being early} \\ c_{free}+\alpha\cdot\tau(t)+\gamma_2\left(\left(t+\tau(t)\right)-t^*\right), \text{for being late} \end{cases} \quad (1)$$

where $c(t)$ denotes travel cost at departure time instant *t*. c_{free} denotes free flow travel cost. $\tau(t)$ denotes travel time (i.e. delay in the bottleneck) at departure time instant *t*. t^* denotes the preferred arrival time. α, γ_1 and γ_2 denote value of travel time, value of early schedule delay and value of late schedule delay respectively.

The travel time for travelers departing at time *t* then is (see [1]):

$$\begin{aligned} \tau(t) &= \frac{D_1(t)}{C}-(t-t_0), t_0 \le t \le t_t \\ \tau(t) &= \frac{D_1(t_t)}{C}+\frac{D_2(t)}{C}-(t-t_0), t_t \le t \le t_e \end{aligned} \quad (2)$$

with $D_1(t)=\int_{t_0}^{t} r_1(x)dx$, for $t_0 \le t \le t_t$, and $D_2(t)=\int_{t_t}^{t} r_2(x)dx$, for $t_t \le t \le t_e$. Here, D_1 and D_2 denote cumulative early departures and cumulative late departures respectively. r_1 and r_2 denote early departure rate and late departure rate respectively. t_0 denotes the start time for the first departure. t_t denotes the transition time. Travelers departing before t_t will arrive early at their destination, while those who depart after t_t will arrive late at the destination. Further, t_e denotes the departure time of the last user experiencing delay. *C* is the deterministic constant capacity rate of the bottleneck.

Due to the equilibrium conditions that no user can reduce his travel cost by unilaterally changing departure times, the cost of travel should be constant for all time instants *t*, which implies that $dc/dt=0$. Then early departure rate and late departure rate in the deterministic case can be derived, see [1].

Figure 1 shows an example with the departure pattern for the case with a total demand of 300 and a deterministic capacity of 10veh/min. All the travelers have an identical PAT at 20. In the picture, several critical time points are indicated. Travelers departing at t_t have the longest travel time but will arrive on time. The travelers departing earlier than t_t will arrive earlier than the desired arrival time t^*, while the travelers departing later than t_t will arrive late at the destination.

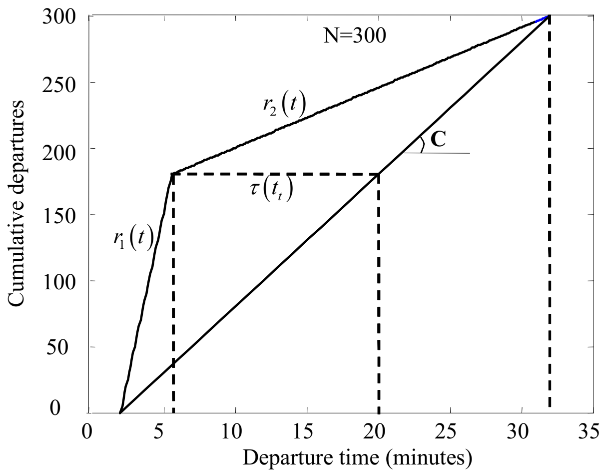

Figure 1. N=300 and C=10veh/min

Figure 2 presents results with different capacities and constant travel demand of N = 200. All the travelers have the same PAT = 50.

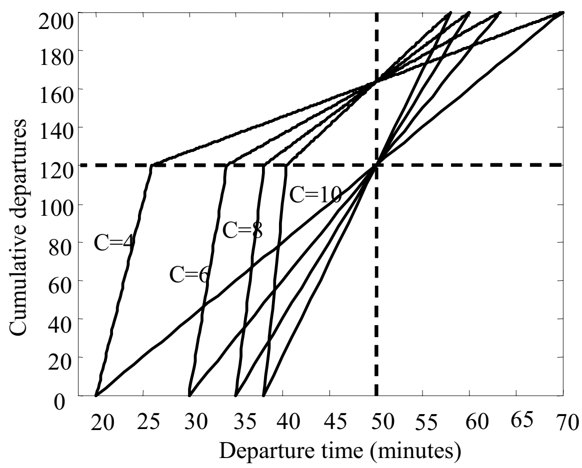

Figure 2. Deterministic cases with different capacity rates C (in veh/min)

It shows that the early departure rate increases with increasing capacity C, while the time duration decreases with increasing C. With increasing C, the starting time for departures shifts to later instants. All the cumulative curves from different C values cross the same two points at PAT = 50, which implies that the number of travelers arriving earlier or later than the PAT are constant and independent of capacities. This can be proved theoretically since it can be derived that:

$$\frac{N_{early}}{N_{late}} = \frac{\gamma_2}{\gamma_1} \tag{3}$$

where N_{early} denotes the number of travelers arriving earlier than the PAT. N_{late} denotes the number of travelers arriving later than the PAT. The ratio of the number of travelers arriving early and late is constant and independent of capacities.

4. Traveler's cost function

Due to the stochastic properties of travel time under randomly degradable capacities, travelers make their departure time choices not only based on the expectation of travel time known from past experiences, but also dependent on the reliability of the travel time during the peak. Travelers are assumed to take the variability of travel time into consideration to guarantee a high probability of arriving on time. We assume that all travelers are perfectly aware of the expected travel time and variability of travel time at all departure times. The travel cost they are assumed to consider comprises the expectation of travel times for time instant t, travel time reliability for time instant t, and punishment for schedule delays. For this paper we now have used the following travel cost function (assuming perfect homogeneity in the traveler population) to model the long term equilibrium for user departure time choice under degradable capacities:

$$\begin{aligned} c^*(t) &= c_{free} + \alpha \cdot E\left(\hat{\tau}(t)\right) + \beta \cdot \sqrt{Var\left(\hat{\tau}(t)\right)} \\ &\quad + \gamma_1 \cdot \left(t^* - \left(t + E\left(\hat{\tau}(t)\right)\right)\right), \ (a) \\ c^*(t) &= c_{free} + \alpha \cdot E\left(\hat{\tau}(t)\right) + \beta \cdot \sqrt{Var\left(\hat{\tau}(t)\right)} \\ &\quad + \gamma_2 \left(\left(t + E\left(\hat{\tau}(t)\right)\right) - t^*\right), \ (b) \end{aligned} \tag{4}$$

where $c^*(t)$ denotes the equilibrium travel cost for departing at time instant t. In the remaining of the paper, whenever a variable has a superscript * , except t^* denoting the preferred arrival time, it denotes a variable at equilibrium state. c_{free} is the free flow travel time. $\hat{\tau}(t)$ is the stochastic travel time which is a flow-dependent and capacity-dependent term. $E\left(\hat{\tau}(t)\right)$ and $\sqrt{Var\left(\hat{\tau}(t)\right)}$ represent the expected travel time and the standard deviation of travel time distribution for departure time t as a reliability cost measure. The fourth term in the cost function represents the schedule delay costs of arriving prior to the preferred arrival time t^* (in Formula (4a)) and of being late respectively (in Formula (4b)). Each departure time t has its own stochastic travel time distribution $\hat{\tau}(t)$, the properties of which are assumed known to the travelers. α , β , γ_1 and γ_2 are parameters, representing the value of travel time, value of travel time reliability, value of schedule delays of being earlier and being late separately. In our analysis, relative values of 1.0, 1.2, 0.8, 1.2 are adopted for α , β , γ_1 and γ_2 .

5. Random capacities

The capacity variation modeled in this paper is caused by relatively minor events and weather conditions. The extreme damages such as earthquakes are assumed not to play a role in day-to-day travel decision making. Day-to-day capacity variations are the major factor leading to stochastic travel time variations over days. Within-day capacity is assumed constant. We assume that stochastic capacity (denoted as $\underline{C}$) is completely exogenous and independent of departure flows. In reality, the capacity is a non-negative stochastic variable changing around a certain mean capacity. For simplicity, we assume capacity to follow a uniform distribution with an upper bound $C_{\max}$ and a lower bound $C_{\min}$. Of course more realistic distributions like weibull can be assumed, however it won't influence the findings. We consider the minimum capacity is proportional to the maximum capacity with a fraction factor θ, (i.e. $C_{\min} = \theta C_{\max}, 0 < \theta < 1$).

The analysis with random capacities will be divided into three regimes ($t_0^* \le t \le t_t^*$, $t_t^* \le t \le t_e^{m*}$, and $t_e^{m*} \le t \le t_e^*$). t_0^* and t_e^* are the time instants for the first departures and last departures at long term equilibrium respectively. t_t^* denotes the transition point in the long term equilibrium. Travelers departing at t_t^* will on the average arrive on time at work. We define t_e^{m*} as another critical time instant at which the maximum capacity intersects with the long term departure pattern at equilibrium, derived from:

$$\left(t_e^{m*} - t_0^*\right) C_{\max} = D_1^*(t_t^*) + D_2^*(t_e^{m*}) \tag{5}$$

where D_1^* denotes cumulative early departures at long term equilibrium state during time period $t_0^* \le t \le t_t^*$. D_2^* denotes the cumulative late departures at long term equilibrium during $t_t^* \le t \le t_e^{m*}$. The first two regimes follow from the different schedule delay functions while the third regime is due to the changed outflow rate distribution. Before t_e^{m*}, the outflow rate distribution is equivalent to the capacity distribution being in our case a uniform distribution with an upper bound $C_{\max}$ and a lower bound $C_{\min}$. After t_e^{m*}, the outflow rate distribution becomes time-dependent, partly following a uniform distribution with a time-dependent upper bound $C_b(\mathrm{t})$ and partly equaling the departure rate. The boundary capacity $C_b(\mathrm{t})$, with which travelers departing at *t* will experience exactly zero delay, is expressed as:

$$C_b(t) = \frac{D_1^*(t_t^*) + D_2^*(t_e^{m*}) + D_3^*(t)}{t - t_0^*} \tag{6}$$

where D_3^* denotes the cumulative late departures at long term equilibrium during $t_e^{m*} \le t \le t_e^*$. On days with capacities larger than $C_b(t)$ and smaller than $C_{\max}$, travelers departing at *t* will experience no delay.

The following properties hold for the assumed capacity distribution:

$$\begin{aligned} E\left(\frac{1}{\underline{C}}\right) &= \int_{\theta C_{\max}}^{C_{\max}} \frac{1}{\underline{C}} \cdot \frac{1}{C_{\max} - \theta C_{\max}} d\underline{C} \\ &= \frac{1}{C_{\max}(1-\theta)} \ln\frac{1}{\theta} \end{aligned} \tag{7}$$

$$\begin{aligned} E\left(\frac{1}{\underline{C}_p(t)}\right) &= \int_{C_{\min}}^{C_b(t)} \frac{1}{\underline{C}_p(t)} \cdot \frac{1}{C_b(t) - C_{\min}} d\underline{C}_p(t) \\ &= \frac{1}{C_b(t) - C_{\min}} \ln\frac{C_b(t)}{C_{\min}}, \quad t_e^{m*} \le t \le t_e^* \end{aligned} \tag{8}$$

$$\begin{aligned} E\left(\frac{1}{\underline{C}^2}\right) &= \int_{\theta C_{\max}}^{C_{\max}} \frac{1}{\underline{C}^2} \cdot \frac{1}{C_{\max} - \theta C_{\max}} d\underline{C} \\ &= \frac{1}{C_{\max}(1-\theta)}\left(\frac{1}{\theta C_{\max}} - \frac{1}{C_{\max}}\right) = \frac{1}{\theta C_{\max}^2} \end{aligned} \tag{9}$$

$$\begin{aligned} E\left(\frac{1}{\underline{C}_p^2(t)}\right) &= \int_{C_{\min}}^{C_b(t)} \frac{1}{\underline{C}_p^2(t)} \cdot \frac{1}{C_b(t) - C_{\min}} d\underline{C}_p(t) \\ &= \frac{1}{C_b(t) - C_{\min}}\left(\frac{1}{C_{\min}} - \frac{1}{C_b(t)}\right) = \frac{1}{C_{\min} C_b(t)}, \quad t_e^{m*} \le t \le t_e^* \end{aligned} \tag{10}$$

where $C_p(\mathrm{t})$ represents the time-dependent uniform distribution of capacities in the range of $C_{\min} \le \underline{C} \le C_b(t)$.

These properties are needed to derive the stochastic temporal travel time distributions.

6. Stochastic bottleneck model

6.1. Mathematical formulations and derivations

In this subsection, we extend Vickrey's deterministic bottleneck model to the case of stochastic capacity of the bottleneck. The cost function in case of stochastic travel times over days is given in Formula (4). The travel time function still holds in the stochastic case. Then the expectation of travel time at departure time *t* can be derived as:

$$\begin{aligned} &E(\hat{\tau}(t)) = E\left(\frac{D_1(t)}{\underline{C}} - (t - t_0^*)\right), && t_0^* \le t \le t_t^* \\ &E(\hat{\tau}(t)) = E\left(\frac{D_1(t_t^*)}{\underline{C}} + \frac{D_2^*(t)}{\underline{C}} - (t - t_0^*)\right), && t_t^* \le t \le t_e^{m*} \\ &E(\hat{\tau}(t)) = (1 - p_c(t)) \cdot 0 + p_c(t) \cdot E(\hat{\tau}_p(t)) \\ &\quad = p_c(t) \cdot E\left(\frac{D_1(t_t^*) + D_2(t_t^{m*}) + D_3(t)}{\underline{C}_p} - (t - t_0^*)\right), \\ &\quad C_{\min} \le \underline{C}_p(t) \le C_b(t),\ t_e^{m*} \le t \le t_e^* \end{aligned} \tag{11}$$

where

$$p_c(\mathrm{t}) = \frac{C_b(t) - C_{\min}}{C_{\max}(1-\theta)}, \quad t_e^{m*} \le t \le t_e^* \tag{12}$$

and $\hat{\tau}_p(t)$ denotes the non-zero stochastic travel times experienced by travelers departing at *t*.

Given our assumptions, a stable long term equilibrium pattern will result, of which the temporal departure flow pattern is deterministic/constant from day to day, thus we have

$$E(D(t)) = D^*(t) \tag{13}$$

where $D^*(t)$ denotes cumulative departures at long term equilibrium state.

Then the expected travel time (i.e. the long-term equilibrium travel time) can be expressed as:

$$E\left(\hat{\tau}(t)\right)=D_1^*(t)E\left(\frac{1}{C}\right)-\left(t-t_0^*\right),\quad t_0^*\le t\le t_t^*$$
$$E\left(\hat{\tau}(t)\right)=\left(D_1^*\left(t_t^*\right)+D_2^*(t)\right)E\left(\frac{1}{C}\right)-\left(t-t_0^*\right),\quad t_t^*\le t\le t_e^{m*}$$
$$E\left(\hat{\tau}(t)\right)=p_c(t)\cdot\left[\left(D_1^*\left(t_t^*\right)+D_2^*\left(t_e^{m*}\right)+D_3^*(t)\right)\cdot E\left(\frac{1}{C_p(t)}\right)-\left(t-t_0^*\right)\right] \tag{14}$$
$$C_{\min}\le C_p(t)\le C_b(t),\ t_e^{m*}\le t\le t_e^*$$

The variances of travel time at departure time t can be derived as:

$$Var\left(\hat{\tau}(t)\right)=D_1^{*2}(t)\cdot\frac{1}{C_{\max}^2}\left(\frac{1}{\theta}-\frac{1}{(1-\theta)^2}\ln^2\frac{1}{\theta}\right),t_0^*\le t\le t_t^*$$
$$Var\left(\hat{\tau}(t)\right)=\left(D_1^*\left(t_t^*\right)+D_2^*(t)\right)^2\cdot\frac{1}{C_{\max}^2}\left(\frac{1}{\theta}-\frac{1}{(1-\theta)^2}\ln^2\frac{1}{\theta}\right),$$
$$t_t^*\le t\le t_e^{m*}$$
$$Var\left(\hat{\tau}(t)\right)=p_c(t)\cdot E\left(\hat{\tau}_p^2(t)\right)-E^2\left(\hat{\tau}_p(t)\right)$$
$$=p_c(t)\cdot E\left(\hat{\tau}_p^2(t)\right)-p_c^{\,2}(t)\cdot E^2\left(\hat{\tau}_p(t)\right),$$
$$t_e^{m*}\le t\le t_e^* \tag{15}$$

According to the equilibrium for user departure time choice, we have $dc^*/dt=0$. Together with the boundary conditions (16), (17) and (18), we can derive the function of early and late departure rates at long term equilibrium state in the stochastic capacity case.

$$c^*\left(t_0^*\right)=c^*\left(t_t^*\right) \tag{16}$$
$$c^*\left(t_0^*\right)=c^*\left(t_e^{m*}\right) \tag{17}$$
$$c^*\left(t_0^*\right)=c^*\left(t_e^*\right) \tag{18}$$

The departure rates at the first two regimes can be directly derived as:

$$\begin{cases} r_1^*(t)=\dfrac{\alpha\cdot C_{\max}}{\dfrac{\alpha-\gamma_1}{(1-\theta)}\ln\dfrac{1}{\theta}+\beta\sqrt{\left(\dfrac{1}{\theta}-\dfrac{1}{(1-\theta)^2}\ln^2\dfrac{1}{\theta}\right)}},t_0^*\le t\le t_t^* \\ r_2^*(t)=\dfrac{\alpha\cdot C_{\max}}{\dfrac{\alpha+\gamma_2}{(1-\theta)}\ln\dfrac{1}{\theta}+\beta\sqrt{\left(\dfrac{1}{\theta}-\dfrac{1}{(1-\theta)^2}\ln^2\dfrac{1}{\theta}\right)}},t_t^*\le t\le t_e^{m*} \end{cases} \tag{19}$$

where r_1^* and r_2^* denote early departure rate and late departure rate at long term equilibrium state during $t_0^*\le t\le t_t^*$ and $t_t^*\le t\le t_e^{m*}$ respectively. It can be seen from Formula (19) that the departure rates for the first two regimes are independent of t, but change according to varied capacity ranges. With higher capacities, departure rates in the first two regimes also will increase.

There is no closed-form expression for late departure rate r_3^* during $t_e^{m*}\le t\le t_e^*$. A finite differential method is used to derive the departure pattern for the time period $t_e^{m*}\le t\le t_e^*$ through the differential equation derived from $dc^*/dt=0$. There are still four unknown variables t_0^*, t_t^*, t_e^{m*}, t_e^*. Because there is no-closed-form solution of r_3^*, we need to initialize one of the variables (we choose t_0^*) satisfying formula (20) and select the value of t_0^* with which the equilibrium cost is the minimum satisfying (21):

$$N=\int_{t_0^*}^{t_t^*}r_1^*(x)\,dx+\int_{t_t^*}^{t_e^{m*}}r_2^*(x)\,dx+\int_{t_e^{m*}}^{t_e^*}r_3^*(x)\,dx \tag{20}$$
$$t_0^*=\arg\min_{t_0^*\le t^*}c^*\left(t_0^*\right) \tag{21}$$

After deriving t_0^*, values for t_t^*, t_e^{m*}, t_e^* can be derived sequentially.

6.2. Results and findings

We present graphical results for the long term equilibrium departure flow patterns under random degradable capacity of the bottleneck. Figure 3 shows the equilibrium departure pattern of the deterministic case using the mean of uniform capacity distribution, and of stochastic capacity case, each for a constant travel demand of 300. In the picture, all the critical times and moments are indicated. All the travelers have the same PAT.

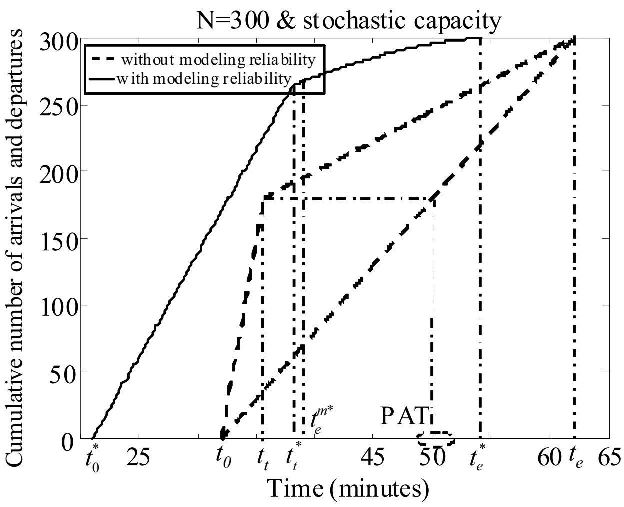

Figure 3. **Departure flows with and without including travel time reliability in the cost function**

It can be seen that the departure patterns are significantly different with and without including travel time reliability in the utility function. Travelers depart earlier when they consider travel time reliability as part of the travel cost, since they attach a safety budget for their travel times. Departure flows are more spread over a longer time period (($t_e^* - t_0^*$) increases). With including travel time reliability in the utility function, the congestion onset at the bottleneck starts earlier and its end is also earlier for the same PAT, compared with the deterministic case.

A capacity line for the equilibrium state doesn't exist in the stochastic case. To be more specific, in the stochastic capacity case the cumulative outflow line is more accurate than the cumulative capacity line, because the outflow rate which equals the departure rate is not always equal to the prevailing stochastic capacity. We define $\hat{A}(t)$ as the stochastic cumulative outflows over days. For explanatory purpose, we define two categories of capacity line at long term equilibrium. One refers to the average cumulative outflow line (i.e. $E\left(\hat{A}(t)\right)$, named category I). The other one refers to a line (named category II) with which $A^*(t+E(\tau(t))=D^*(t)$ holds. $A^*(t)$ denotes the cumulative outflows at equilibrium. Figure 4 shows the cumulative departures and the two categories of cumulative outflow lines at equilibrium.

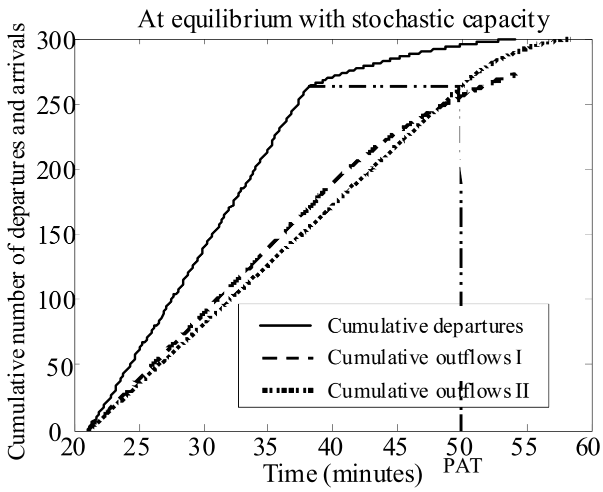

Figure 4. Cumulative departures and two categories of cumulative outflow lines at long term equilibrium

Figure 5 shows the expected travel time at long term equilibrium with stochastic capacities and the travel time at deterministic case. On average, travel times are shorter in the stochastic case. Total travel time is less when including travel time reliability in the utility function. However the total travel cost increases by the extra cost from experienced travel time variability.

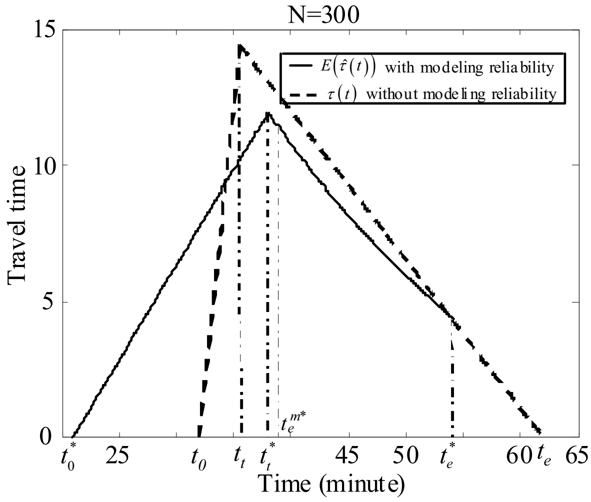

Figure 5. Expected travel times with stochastic capacities and travel times with constant capacity

Figure 6 presents the long term equilibrium cost by components. It is noticed that the travel time variability is increasing with departure time and then decrease a bit. It implies that during the peak the later a traveler departs, the larger travel time variability he/she might experience. That's also a reason that travelers depart earlier in order to reduce their travel time variability. One important finding is that travel time variability is not proportional to the expectation of travel times as one might assume at first sight.

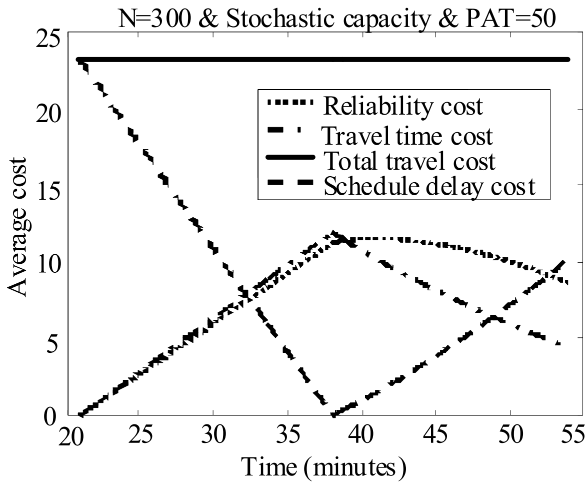

Figure 6. Long term equilibrium costs by component

7. Conclusions and future works

The deterministic bottleneck model of Vickrey has been extended with random capacities. To facilitate an analytical treatment a simple uniform capacity distribution has been adopted. Travelers are assumed to value travel time variability as an extra travel cost. Consideration of stochasticity in the bottleneck capacity and consequently in travel and arrival times lead to significant shifts in the temporal demand pattern. Travelers will compensate for the uncertainty by traveling earlier. Travel time variability is increasing with departure time and then decrease a bit. The later a traveler departs, the larger travel time variability he/she might experience. This is also a reason that travelers attempt to depart early in order to minimize travel time variability. It is found that travel time variability (standard deviation of travel time distribution) is not proportional to the expected travel time as one might assume at first sight.

Our theoretical results are supported by empirical findings of delays at bottlenecks. Increasing the bottleneck capacity for example has shown at several places a shift in demand towards later departure times, also known under the term 'return-to-the-peak' phenomenon.

The departure time choices and resulting departure pattern under stochastic capacities modeled in this paper is very valuable for correctly assessing the network performance and evaluating dynamic traffic management measures.

Future work will address the extension of the homogeneous travelers case to the heterogeneous case with subgroups having different values of time and values of schedule delay. Also a distribution of PAT can be modeled due to travelers with different trip purposes having different preferred arrival times. Part of these work has been done, for more information, see [7]. Within-day capacity variations and fluctuating travel demand are also major factors leading to travel time variability, which will be investigated in future work.

8. Acknowledgement

This study is supported by the Transport Research Centre Delft of the Delft University of Technology and by the Next Generation Infrastructures research program in the Netherlands.

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H. Li Hao Li received her MSc degree in Transportation and Planning with honors in 2005 at Delft University of Technology. She is a PhD Candidate in the transportation and planning department at Delft University of Technology. Her main research interests are transport network modeling, network design, properties of stochastic route choice set generation, road pricing, and network reliability. She has (co)authored several papers for international conferences, some of which are to be published in journals. She has won the best scientific paper award at 14th World Congress on ITS, Beijing, China, 2007.

P. H. L. Bovy Piet Bovy received his MSc degree in Civil Engineering at the University of Technology RWTH Aachen, Germany in 1967 and finished his PhD in transportation at Delft University of Technology in 1990. He has been full professor in Transportation Planning & Engineering at the Delft University of Technology since 1993, as well as director of Transportation Research at Delft University of Technology, and scientific director of the Netherlands Research School TRAIL for Transport Infrastructure and Logistics from 1996 to 2004. He has co-authored two books and wrote the first book (1990, together with E. Stern) on route choice in transport networks. Piet is editor and co-editor of several books and is member of the editorial board of several journals and international

conferences. He has numerous scientific publications in journals, books and conference proceedings.

M. C. J. Bliemer Michiel Bliemer received his MSc degree in Econometrics with honors in 1996, and finished his PhD thesis in traffic engineering in 2001. Currently he is associate professor in transport modeling at Delft University of Technology and adjunct professor at the University of Sydney. His main research interests are dynamic network modeling, road pricing, discrete choice models, and experimental design. Michiel has (co)authored several papers in top tier transport journals in these areas, has produced commercial software for dynamic traffic simulation (INDY), is a member of several committees such as at the US Transportation Research Board, and is associate editor of the Journal of Choice Modelling.

- *Received date: December 15 2007*
- *Received in revised forms: March 7 2008 , April 25 2008*
  August 26 2008
- *Accepted date: August 26 2008*
- *Editor: Ching-Yao Chan*