

Evaluation on Effect of Arrival Time Information Provision Using Transit Assignment Model

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Using capacity-constrained transit assignment model which takes the common lines problem into account, this paper proposes the method for analyzing the effect of arrival time information while comparing with that of the other hardware improvements.

The result suggests that arrival time information provision contributes to as much effect as hardware improvements for the whole network. However, the effect is different among OD pairs: information provision brings more effect at the upstream stations whereas the little effect can be shown at the downstream stations. We also find that effect does not directly contribute to the mitigation of congestion.

Keywords: Transit Assignment, Common Lines Problem, Arrival Time Information Provision

1 Introduction

The problem of full transit vehicles is experienced daily by passengers throughout the world. In order to mitigate the congestion, the transit operators have focused on hardware improvements such as increasing the vehicle capacity, increasing the service frequency, and expanding the platform. In addition, since ITS technology has been developed, they are providing the information about when the next vehicle will come throughout many devices such as electronic signboard on the platform, internet, mobile phone, and so on. However, they do not pay much attention for how information provision brings effect on passengers' route choice behavior. If passengers' behavior is changed by information provision, the congestion mitigation level on the network will also be changed. Therefore, it is important to evaluate how arrival time information provision affects passengers' route choice behavior. In order to describe the passengers' route choice behavior, many researchers so far proposed transit assignment model. In general, there are two main differences between private traffic assignment and public traffic assignment, which is the common lines problem and capacity restrained.

1.1 Common Lines Problem

Waiting time for each passenger at each platform

due to service frequency exists in public transport systems. Therefore, passengers might wait a long time to get on a line of the shortest travel time, that is to say, they cannot always get to their destination earliest even when they take a line of the shortest travel time. Sometimes the fastest way to get to their destination is to take the first vehicle to arrive among their "attractive" lines. This issue is referred to as the common lines problem [1]. Common lines problem is defined as the problem to find the optimal attractive lines among all lines getting to their destination. Suppose passengers and vehicles arrive randomly, the probability that passengers choose line i , p_i , and the expected travel time to their destination, T , are calculated as follows:

$$p_i = \frac{f_i}{\sum_{k \in K} f_k} \quad (1)$$

$$T = \frac{1 + \sum_{k \in K} f_k t_k}{\sum_{k \in K} f_k} \quad (2)$$

where,

K : Set of attractive transit lines,

t_k : In-vehicle time on transit line $k \in K$,

f_k : Frequency on transit line $k \in K$.

Nguyen and Pallottino [2] defined attractive lines as *hyperpath*. Then, Spiess and Florian [3] combined the common lines problem and the equilibrium assignment problem in a linear programming framework, assuming passengers choose a hyperpath to minimize their expected travel time and take the first vehicle to come which belongs to the hyperpath.

To find the solution, a non-linear mixed integer program with a total travel time objective plus flow conservation and non-negativity constraints was first formulated and converted into a linear program. They also constructed a model whose link costs, t_k , is monotonically increasing function of link flow to reflect the greater disutility associated with crowded vehicles. However, as they acknowledge, this extension does not allow for full vehicles.

1.2 Capacity Restrained

According to queuing theory, waiting time at the station becomes infinity when passengers' arrival rate reaches vehicles' capacity. De Cea and Fernandez [4] showed a model which approximates waiting time at stations as a BPR-type function. The problem of this approximation is that waiting time remains finite even when passengers' arrival rate reaches vehicles' capacity. Then, Cominetti and Correa [5] present a framework for congested transit assignment that can incorporate congestion functions obtained from queuing models. However, their model is mathematically too strict to solve in a complicated network. Since those two models are based on queuing theory, it is assumed that the arrival rates of passengers are stable and less than capacity during the modeled period. Therefore, it is impossible to represent passengers' arrival rate exceeding vehicles' capacity in a short period, which could happen during peak hours.

On the other hand, Schmöcker *et al.* [6] loads passengers explicitly considering capacity constraints. The model is called Cap-Con and it includes the risk of failing to board in the cost functions and therefore in the choice process. If passengers are risk-averse, they would seek to use lines with low possibilities of failing to board. Then, Kurauchi *et al.* [7] implemented common lines problem into the model proposed by Schmöcker *et al.* [6], called CapCon-CL. As presented later, they adopted the idea of the *hyperpath* and transformed a transit network into a graph model to consider the capacity restrained.

All the models referred so far assume that passengers cannot get information about when next vehicle comes. Therefore, we modified the model proposed by Kurauchi *et al.* [7] to consider the arrival time information provision. This paper attempts to evaluate the effect of arrival time information on the platform onto passengers' flow, comparing with the effect of capacity or frequency improvement.

2 Notations

L : Set of transit lines,

A : Set of arcs,
 A_p : Set of arcs on hyperpath p ,
 I_p : Set of nodes on hyperpath p ,
 R : Set of origin nodes,
 S_p : Set of stop nodes on hyperpath p ,
 E_p : Set of failure nodes on hyperpath p ,
 V_p : Set of elementary paths on hyperpath p ,
 D_s : Set of failure arcs destined to s ,
 U_l : Set of platforms on transit line l ,
 $OUT_p(i)$: Set of arcs that lead out of node i on hyperpath p ,
 f_l : Frequency of service on transit line $l \in L$,
 z_l : Capacity of transit line $l \in L$,
 t_a : Travel time on arc $a \in A$,
 τ_p : Arc split probability for each node on hyperpath p
 q_i : Probability that a passenger fails to board at node $i \in E$,
 WT_{ip} : Expected waiting time at stop node i on hyperpath p ,
 m_{rs}^* : Minimum cost from the origin of hyperpath $p(r)$ to the destination (s),
 θ : Risk parameter for fail-to-board,
 δ_{al} : Takes 1 is if arc a is included in l and 0 otherwise,
 ε_{il} : Takes 1 if elementary path l traverses node i and 0 otherwise,
 λ_{lp} : Probability of choosing any particular elementary path l of hyperpath p ,
 α_{ap} : Probability that passengers on hyperpath p traverses arc a ,
 β_{ip} : Probability that passengers on hyperpath p traverses node i ,
 Ω : Set of feasible hyperpath flows which satisfy flow conservation,
 w_{kl} : Stopping arc of line l on platform k ,
 b_{kl} : Boarding demand arc on line l on platform k ,
 h_{kl} : Failing node on line l on platform k ,
 \mathbf{x} : A vector of arc flows,
 \mathbf{y} : A vector of hyperpath flows,
 \mathbf{q} : A vector of fail-to-board probabilities,
 P_{rs}^* : Optimal hyperpaths set connecting r and s .

3 Network Representation

In order to consider the capacities of the transit lines together with the common lines problem, a transit network shown in Figure 1(a) is transformed into a graph model as Figure 1(b). Bold numbers on transit line in Figure 1(a) represents a travel time between stations. The idea follows line-specific arcs proposed by Fearnside and Draper [8] so that the capacity of transit lines can be easily considered, whereas stop nodes are included to explain the common lines problem in the manner that Nguyen and Pallotino [2] proposed. Furthermore, to explain a failure to board,

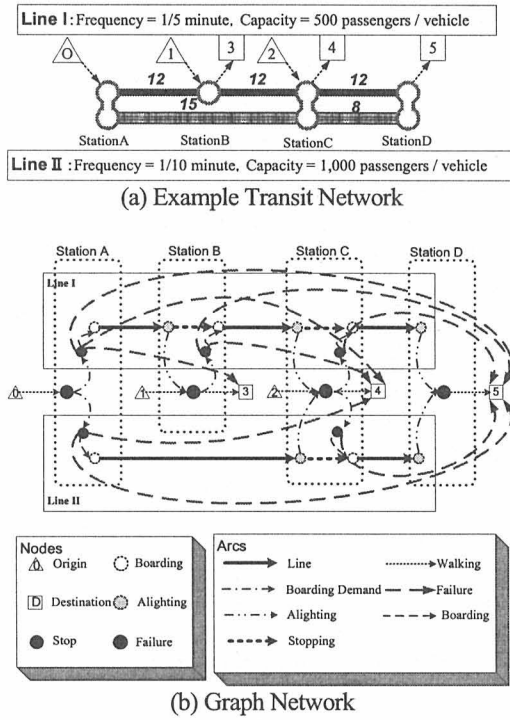


Figure 1 Network Representation

failure node and arcs are implemented. Nodes are categorized into six types and arcs are categorized into seven types.

An *origin node* represents a trip start node. It has no predecessors and at least one successor. A *destination node* represents a trip end node. It has no successors and at least one predecessor. A *stop node* represents a platform of a station. Any transit lines stopping at the same platform are connected via boarding demand arcs, failure nodes and boarding arcs. At stop nodes, passengers are assigned to any of the attractive lines in proportion to the arc transition probabilities. A *boarding node* is a line-specific node at the platform where passengers board. An *alighting node* is a line-specific node at the platform where passengers alight. A *failure node* is a node that explains failure to board. When transit line capacity is exceeded if all passengers get on board, some of them are forced to use the failure arc. It has one predecessor arc from the stop node and (1+ number of destinations) successors. One arc is connected to the corresponding boarding node and the others are connected to each destination node.

A *line arc* represents a transit line connecting two stations. A *boarding demand arc* denotes an arc connecting the stop node to the failure node. The flow on this arc represents the boarding demand for the transit line from a specific platform. An *alighting arc* denotes an arc from an alighting node to a stop node. A *stopping arc* denotes a transit line stopping on a platform after the passengers alight and before

the passengers board. This arc is created to express the available capacity on the transit line explicitly. A *walking arc* connects an origin to a platform or a platform to a destination. A *failure arc* denotes the demand that failed to board. This arc is a dummy arc to set to satisfy flow conservation and excess demand is sent hypothetically to its respective destination by this arc. A *boarding arc* is an arc connecting a failure node to a boarding node.

Our network representation is possible to examine a lot of situations, such as increasing the capacity or frequency of a certain line. Also, it is possible to investigate the case of existing different types of train, local train and express train, if we prepare the line arc for each types of train.

4 Capacity Constraints Transit Assignment Model with Common Lines (CapCon-CL)

4.1 Hyperpath

To obtain an attractive set of transit lines that minimizes expected travel time, we adopt the idea of the *hyperpath* proposed by Nguyen and Pallottino [2]. The hyperpath connecting an origin r to a destination s is defined as sets of stops, arcs and arc transition probabilities $H_p = (I_p, A_p, T_p)$, where H_p is a hyperpath connecting r to s , if:

- H_p is acyclic with at least one arc;
- node r has no predecessors and s no successors;
- for every node $i \in I_p - \{r, s\}$, there is a path from r to s traversing i , and if node $i \notin R$, then i has at most one immediate successor;
- the vector, τ_p , contains the arc split probabilities, which satisfies

$$\sum_{a \in OUT(i)} \tau_{ap} = 1, \forall i \in I_p, \quad (3)$$

and

$$\tau_{ap} \geq 0, a \in A. \quad (4)$$

4.2 Arc Split Probability

Where there are several arcs leading out of nodes on a hyperpath, traffic is split according on τ_{ap} . As shown in Figure 1, traffic may be split at either stop, failure or alighting nodes.

4.2.1 Stop nodes

When we adopt the following assumptions regarding the common lines problem:

- Passengers arrive randomly at every stop node, and always board the first arriving carrier of their choice set; and
- All transit lines are statistically independent with given exponentially distributed headways, and mean equal to the inverse of line frequency.

Then τ_{ap} is calculated as follows:

$$\tau_{ap} = f_{l(a)} / F_{ip}, \forall i \in S_p, \forall a \in OUT_p(i) \quad (5)$$

$$F_{ip} = \sum_{a \in OUT_p(i)} f_{l(a)} \quad (6)$$

4.2.2 Failure nodes

When q_i is not zero, some passengers fail to board. Traffic is split according to the failure-to-board probability q_i at failure nodes.

$$\tau_{ap} = \begin{cases} 1 - q_i & \text{if } a \notin D_s, \forall i \in E_p \\ q_i & \text{if } a \in D_s \end{cases} \quad (7)$$

4.2.3 Alighting nodes

There may be several arcs leading out of alighting nodes. However, more than one waiting and alighting arcs are never included in an optimal hyperpath because getting on the next train on the same line as one gets off is apparently irrational.

4.3 The Cost of Hyperpaths

In this paper, the cost of a hyperpath consists of three elements, expected in-vehicle time, expected waiting time, and the implicit cost associated with the risk of failing to board. Before discussing the cost of hyperpath, the node and arc transition probabilities are defined. Let V_p denote a set of all elementary paths in H_p , and δ_{al} is 1 if arc a is included in l , otherwise 0. Let λ_{lp} denote the probability of choosing any particular elementary path l of hyperpath p , then clearly:

$$\lambda_{lp} = \prod_{a \in A_p} \tau_{ap}^{\delta_{al}}, \forall l \in V_p, \quad (8)$$

$$\sum_{l \in V_p} \lambda_{lp} = 1. \quad (9)$$

Let β_{ip} denote the probability of traversing node i , namely the node transition probability, and ε_{il} is 1 if elementary path l traverses node i . Then β_{ip} can be calculated from λ_{lp} as follows:

$$\beta_{ip} = \sum_{l \in V_p} \varepsilon_{il} \lambda_{lp}, \quad \forall i \in I_p. \quad (10)$$

Similarly α_{ap} , probability that traffic traverses arc a , i.e., arc transition probability, can be calculated as follows,

$$\alpha_{ap} = \sum_{l \in V_p} \delta_{al} \lambda_{lp}, \quad \forall a \in A_p. \quad (11)$$

Using node and arc transition probabilities, the cost

of hyperpath p , g_p , can be written as follows:

$$g_p = \sum_{a \in A_p} \alpha_{ap} t_a + \sum_{k \in S_p} \beta_{kp} \cdot WT_{kp} - \theta \ln \left(\prod_{k \in E_p} (1 - q_k)^{\beta_{kp}} \right) \quad (12)$$

The first term and the second term represent the travel time for the arcs and the expected waiting time at stop nodes respectively. WT_{ip} is expected waiting time at stop node i of hyperpath p , which can be calculated as follow:

$$WT_{ip} = 1 / F_{ip}, \quad (13)$$

$$F_{ip} = \sum_{a \in OUT_p(i)} f_{l(a)} \quad (6)$$

The third term represents the cost associated with the risk of failing to board. Parameter for risk of failing to board, θ , denotes risk averseness. If $\theta \rightarrow \infty$, then passengers are absolutely risk averse, and they are not interested in travel time and expected waiting time; when $\theta = 0$ passengers do not care about failing to board.

4.4 Mathematical Formulation

Let us assume that passengers use a hyperpath of minimum cost. The cost of a hyperpath is a function of the failure-to-board probability for each transit line on each platform. On the other hand, failure-to-board probability depends on boarding demand, passengers already on board, and transit line capacity, which in turn depends on the failure-to-board probability. Therefore, this can be regarded as a fixed point problem which defines the equilibrium. Eventually, the equilibrium is given by the solution to the following fixed point problem:

Find (y^*, q^*)

such that

$$y^* \cdot u(y^*, q^*) = 0, u(y^*, q^*) \geq 0, y \in \Omega \quad (14)$$

$$q^* \cdot v(y^*, q^*) = 0, v(y^*, q^*) \geq 0, \forall 0 \leq q \leq 1 \quad (15)$$

where,

$$u_p(y^*, q^*) = g_p(y^*, q^*) - m_{rs}^* \quad (16)$$

$$v_{kl}(y^*, q^*) = f_l z_l - x_{w_{kl}} - (1 - q_{h_{kl}}) x_{b_{kl}}, \quad \forall k \in U_l, l \in L \quad (17)$$

In Equation (14), u denotes a vector of cost difference between $g_p(y^*, q^*)$ and minimum cost from the origin of the hyperpath p (r) to the destination (s), therefore, Equation (14) represents the user equilibrium condition. In Equation (15), v denotes the vector of vacancies on the line arc on line l from platform k , therefore, Equation (15) represents the capacity constraint condition. As we see in Equation (14) and (15), the model proposed here is static one.

Therefore, although it is difficult to consider the dynamic schedule in this model, this model needs less time to calculate than dynamic model or simulation model, which is one of the advantages especially when calculating in a large network.

The existence of a fixed point is intuitive since any excess demand simply implies non-zero failures to board. However, because of the non-linear relationship in Equation (15) there is a possibility of multiple fixed points. This fixed point problem can be solved by combining the method of successive averages and absorbing Markov chains (Kurauchi *et al* [7]).

4.5 Passenger Behavior with Arrival Time Information

So far we assumed that arrival time information is not available for passengers. However, since many transit companies provide arrival time information, passengers can choose the transit line which reaches the destination station earliest. Therefore, it may happen that passengers can arrive at their destination earlier if they pass the first train, and use the subsequent one. In this section, we further explore the passenger behavior under information provision. Let us assume that passengers can know when the next train of all lines will come at the platform and the information is strictly accurate. For the sake of solving analytically, we consider that passengers can choose their route from less than two lines at the platform.

Given that waiting time for line 1 is w_1 , the probability that passengers get on line 1 is calculated as follows:

$$\begin{aligned} \Pr(i=1|w_1) &= \Pr(t_1 + w_1 < t_2 + w_2 | w_1) \\ &= \Pr(w_2 > t_1 - t_2 + w_1 | w_1) \\ &= \int_{\max\{0, t_1 - t_2 + w_1\}}^{\infty} g_2(x) dx \end{aligned} \quad (18)$$

where,

- t_k : In-vehicle time on transit line k ($k=1,2$),
- w_k : Waiting time for line k ,
- $g_k(x)$: Probability that waiting time of line k is x .

As w_1 follows the distribution, the probability of choosing line 1, τ_1 , can be written as follow:

$$\begin{aligned} \tau_1 &= \int \Pr(i=1|x) dx \\ &= \int \left\{ \int_{\max\{0, t_1 - t_2 + x\}}^{\infty} g_2(y) dy \right\} \cdot g_1(x) dx \end{aligned} \quad (19)$$

The probability of choosing line 2 can be calculated similarly. Then, the expected waiting time is calculated as follows:

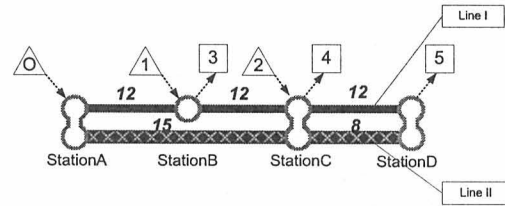


Figure 2 Example Network

Table 1 Scenario Cases

	Information Provision	Capacity (passenger/min.)		Time Headway (minute)	
		Line I	Line II	Line I	Line II
Case 1	No	150	150	5	10
Case 2	Yes	150	150	5	10
Case 3	No	200	150	5	10
Case 4	No	150	200	5	10
Case 5	No	150	150	3	10
Case 6	No	150	150	5	8

$$\begin{aligned} T &= \int \left\{ \int_{\max\{0, t_1 - t_2 + x\}}^{\infty} g_2(y) dy \right\} \cdot g_1(x) (t_1 + x) dx \\ &+ \int \left\{ \int_{\max\{0, t_2 - t_1 + x\}}^{\infty} g_1(y) dy \right\} \cdot g_2(x) (t_2 + x) dx \end{aligned} \quad (20)$$

In case the waiting time distribution, $g_k(x)$, is exponentially distributed, the arc split probability, τ_1 and τ_2 , can be calculated as follows:

$$\tau_1 = \frac{f_1}{f_1 + f_2} \exp(-f_2 \cdot DT), \quad \tau_2 = 1 - \tau_1 \quad (21)$$

where,

$$DT = t_1 - t_2 \geq 0 \quad (22)$$

We must mention here that all the lines are possible to be chosen since random arrival of vehicles are assumed. Also, the expected waiting time WT_p is calculated as follows:

$$WT_p = \frac{1 - f_1 \cdot DT}{f_1 + f_2} \exp(-f_2 \cdot DT) + \frac{1 - \exp(-f_2 \cdot DT)}{f_2} \quad (23)$$

Compared Eq.(5) to Eq.(21), it can be said that arrival time information causes concentration to the paths of shorter travel time. This is because there is a possibility that the faster train which leaves the origin station later may reach the destination station earlier. Also, it is easily proved that under no congestion, expected travel time to the destination is reduced due to information provision.

5 Numerical Example

5.1 Test Conditions

The model described in the previous section is applied to a simple test network with four stations and two parallel lines as shown in Figure 2. The travel times between stations are shown in the figure and the travel demand between each OD pair is set to

increases because passengers sometimes do not take the vehicle of Line I that comes first but wait for Line II. In conclusion, information provision helps reducing the number of passengers failing to board, whereas it may cause congestion on the platform.

5.4 Comparing the Effect of Information with Hardware Improvement

At the previous section, we have found that the transit line I is rather congested under no information provision and that arrival time information is effective to avoid failing to board. In this section, we will compare the effect of information provision with hardware improvement from the aspect of congestion mitigation. We evaluate two measures as a hardware improvement in this paper; increasing the capacity of lines and increasing the frequency of the lines.

5.4.1 Increasing the capacity of lines

Table 3 represents the cost of hyperpath. As shown in the table, all passengers can get benefit by increasing the capacity of Line I (Case 3), whereas no passengers can get benefit by increasing the capacity of Line II (Case 4). Therefore, it is effective to increase the capacity of more congested line for reducing the cost of hyperpath. Also, in this case, the effect of increasing the capacity of Line I (Case 3) is almost equal to that of providing information (Case 2) in terms of the cost of hyperpath. However, the benefit is not equally distributed for all OD pairs: for OD pairs departing from the station where the line starts, information provision can reduce the cost more than increasing the capacity, but for OD pairs originated from the interim stations of the line, information provision cannot produce large benefit as increasing the capacity. Figure 6 illustrates the values of connectivity reliability. From Figure 6(a), information provision (Case 2) cannot obtain so much benefit as increasing the capacity (Case 3). Therefore, it can be said that compared with increasing the capacity of congested line, information provision can expect as much effect in terms of cost, but it cannot expect so much effect in terms of connectivity reliability.

5.4.2 Increasing the frequency of lines

Looking at Table 3, in terms of cost reduction, increasing the frequency of Line I or Line II is effective in total. Also, the effect of increasing the frequency is almost equal to that of information provision. However, judging from the connectivity reliability, the effect is totally different. Looking at Case 5 and Case 6 of Figure 6, increasing the capacity of Line I contributes to improve the connectivity reliability, whereas increasing the

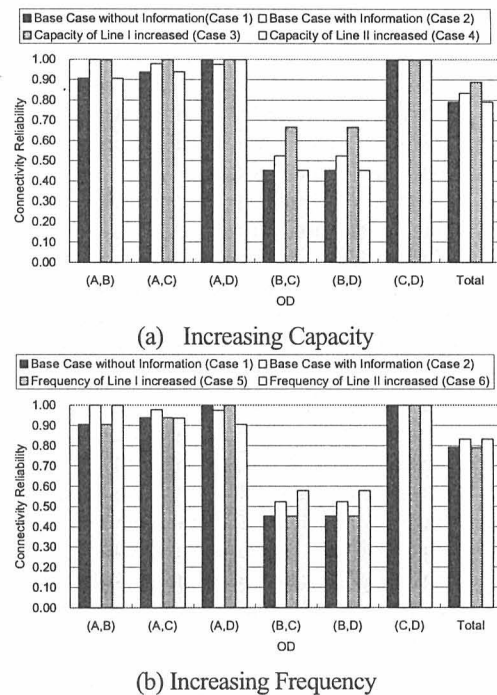


Figure 6 Connectivity Reliability

capacity of Line II does not. Therefore, it is better to increase the frequency of less congested line. Moreover, it is interesting to say that the connectivity reliability of OD pairs originated from station B also improves when increasing the frequency of Line II (Case 6).

Next, we will compare the effect of information provision (Case 2) with that of increasing the frequency of Line II (Case 6). The average of the connectivity reliability is almost equal for both cases. However, looking at the results of each OD pair, the effect is also different: for the OD pairs originated from the station where the line starts, information provision produces more benefit than increasing the frequency, but for the OD pairs originated from interim stations, increasing the frequency provides more benefit.

6 Conclusions

This paper evaluated the effect of arrival time information using the transit assignment model. From the simple case studies, we obtained following findings:

- (a) Since arrival time information leads to improve the connectivity reliability, it contributes to mitigate the congestion for the whole network. However, it is possible that arrival time information encourages passengers to use the shorter in-vehicle time line and the connectivity

reliability of some OD pairs is decreased as a result.

- (b) Arrival time information may conduce to increase the expected waiting time at the platform, which may result in increase the congestion at the platform.
- (c) In terms of hyperpath cost reduction, arrival time information can expect almost the same effect as hardware improvement for the whole network. However, the effect is different between OD pairs: arrival time information provision can expect more effect than hardware improvement at the origin station of the lines whereas less effect at the interim stations.
- (d) In terms of improvement of the connectivity reliability, arrival time information can expect almost the same effect as increasing the frequency of lines for the whole network, whereas it can not expect so much effect as increasing the capacity of the lines. Moreover, the effect is not equally distributed among the OD pair.

It must be mentioned that the conclusion listed above cannot be generalized since the result will be affected by the network or demand. However, the method proposed here can be applied even when the network or demand is changed. Also, we found such a case that the cost of hyperpath is reduced due to the frequency increment but the connectivity reliability remains the same. Therefore, when evaluating the congestion mitigation effect of information provision or service level improvement, it is important to use not only the cost of hyperpath but also the connectivity reliability.

For further investigations, the model presented here needs to be applied to a realistic network. Also, the evaluation index for transit operator, such as operation cost, should be considered.

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