Modeling of Road Pricing considering Local Emissions of Road Transportation Network

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Road pricing from an economic point of view has largely been motivated by the time costs that each road user imposes on other road users. For many years road pricing has been attracting considerable attention of analysts and policymakers as a means of relieving urban traffic congestion and recently for local traffic air-pollution problems. In this paper, an optimum model is proposed considering local traffic emissions and illustrated with hypothetical network. Davidson time-flow function and vehicle speed-emission factor relationship is considered. Augmented lagrangean algorithm is proposed for solution and is validated by comparing the pricings with that of descriptive approach using UE model. It is found that the model estimates quite accurate solution.

Optimum tolls, Road pricing, local emissions, External costs, Environmental externalities

1. Introduction

Road pricing from an economic point of view has largely been motivated by the time costs that each road user imposes on other road users, since the classical discussion of external costs by Pigou (1920).There have been extensive studies in road pricing (Yang Hai and Huang 2005; Johansson, 1997; May, 2000) as an efficient approach to internalize externalities such as congestion, air pollution. Congestion pricing schemes do not necessarily lead to less traffic emissions (May, 2000; Yin, 2006). In recent years, externalities with particular attention to local air-pollution problems associated with urban transportation (Button and Verhoef, 1998) are seriously considered by policy makers and government agencies. Transportation is responsible for approximately 50% of the emissions of nitrogen oxide and 90% of the carbon monoxide (Nagurney, 2000). Nagurney (2000) demonstrated that the network topology, cost structure, as well as the travel demand structure must be taken into consideration in any policy system aimed towards the reduction of emissions due to motor vehicles. Hence there is need to model road pricing considering local emissions to avoid traffic related emission problems in urban areas and also consider all the relevant parameters of network to keep local emissions within the permissible levels. In this paper, a model is proposed considering local traffic emission standards for selected routes and illustrated with hypothetical network. Davidson time-flow equation is considered in the model which estimates accurate speeds over BPR or other functions for flows below capacity (Singh, 1999). The speed-emission factor relationship is introduced in the model for emission estimates to further determine the link environmental capacities that can be allowed in the network. Here, the focus is on road networks with fixed

O-D demand and single vehicle type with homogenous users is considered. Further, only the static deterministic equilibrium solution is considered. This paper consists of 6 chapters. Chapter 1 discusses the general problems of local emissions and its importance, chapter 2 discusses the previous studies related to congestion pricing models and emission related pricing studies, chapter 3 discusses the model including the parameters and formulation, chapter 4 discusses the solution algorithm, chapter 5 discusses the model illustration with hypothetical network and validation by comparing the road pricings of optimum approach and descriptive approach and finally discusses the conclusions in chapter 6.

2. Previous studies

In view of modeling road pricing considering local emissions, it is necessary to overview the existing congestion pricing models and emission related pricing studies. Motorists typically select routes that minimize their travel time or generalized cost. This may entail traveling on longer but faster routes. This raises questions concerning whether traveling along a longer but faster route results in energy and/or air quality improvements (Kyoungho et.al.,2008).The theoretical background of road-use pricing has relied upon the fundamental economic principle of marginal-cost pricing, which states that road users using congested roads should pay a toll equal to the difference between the marginal-social cost and the marginal-private cost in order to maximize the social surplus (Verhoef,1996). Marginal-cost pricing theory or the first-best pricing in the literature by pigou and followers (Walters,1961, et.al.) were developed based on the demand-supply (or performance) curves for the standard case of homogeneous traffic stream moving along a given uniform stretch of road, such as an expressway, connecting given entry and exit points. In the case of homogeneous users, the first-best congestion pricing theory is established in general traffic networks. In line with this theory, a toll that is equal to the difference between the marginal social cost and marginal private cost is charged on each link so as to internalize the user externalities and thus achieve a system optimum flow pattern in the network (Beckmann, 1965 et.al). Investigations have been conducted on how this classical economic principle would work in a general congested road network with multiple vehicle types and link flow interactions, with queing (Yang and Huang, 1998), and in a congested network in a stochastic equilibrium (Yang, 1999). Moreover, Bellei et al. (2002) developed a variational inequality model for network pricing optimization in a multi-user and multi-modal context. Liu and Boyce (2002) presented a variational inequality formulation of the marginal-cost pricing problem for a general transportation network with multiple time periods. Yang et.al (2005) formulated system optimum problem for a general congested network keeping a capacity constraint and indicated that environmental problem can be dealt similarly by keeping the constraints, but did not actually modeled considering vehicle emission factors.

Prudhomme (2005) estimates the environmental benefits are about 10% of the congestion benefits for London congestion pricing. Johansson (1997) shown that a road user should pay a charge to the increased emissions and fuel consumption of other road users in addition to their own emissions. Johansson (2006) further shown a model that an optimal first-best road charge should not only be differentiated with respect to factors that affect the direct external environmental and time costs from the road-user himself. But also include indirect effects that others' cars will be more polluting when congestion increases. Link interaction was not considered in that model where the link speed is not independent. Yin (2006) has shown a model for emission charging that minimizes the total emissions with an example of CO emission factor. However the charges that minimize the total emissions may not keep the emissions below the permissible levels on individual selected locations based on their sensitiveness of area particularly when selected local air pollutant that may not require minimizing in non-sensitive areas. Kyoungho et.al.(2008) indicated that different emission- and/or energy-optimized assignments should be recommended for each pollution type and fuel consumption and CO_2 emission- and energy-optimized assignment is identical to the SO assignment.

Therefore, due to growing importance of local pollution problems and very little research included it, in this paper local air pollutant (NOx) is considered for the optimum model. This model is identical to Yang et.al congestion pricing model but with individual link emission constraints using NOx speed-emission factor relationship.

3. Model formulation

The main objective is to formulate optimum Environmental Road Pricing model that keep the local emissions below the permissible emissions on selected locations which are selected based on sensitivity of the location to reduce health impacts. Many models were formulated for congestion pricing in the literature (Yang et.al, 2005 and others) and may have some indirect benefits of emissions reductions. However, congestion pricing may not always have emission reductions below permissible levels in all selected locations due to the minimization of system costs alone. In the same way, the minimization of total emissions alone may not keep the individually selected location emissions below the permissible emissions. The minimization of total emissions is the case with global emissions such as CO_2.

System optimization is well received by analysts and policy makers and hence Environmental Road Pricing combined with Congestion Pricing (CP+ERP) is formulated which is the combination of Congestion Pricing (CP) and Environmental Road Pricing (ERP). These CP or ERP options are shown in the end separately with few changes in the CP+ERP formulation. Formulation optimizes the solution that satisfies the emission constraints. Various parameters such as time-flow equation, vehicle mileage, vehicle emission factors in addition to network parameters are considered in the model.

3.1 Assumptions

Consider fixed demand with route choice only in the model. Consider a single vehicle category with homogeneous users and single pollutant. Both travel time costs and fuel consumption costs are considered in the model. Consider linear relationship between vehicle average speed and vehicle mileage.

3.2 Parameter description

Consider a network with O-D pairs W, A set of links, Q set of link flows and R_w set of routes in each OD pair w. Consider fixed demand d_w (veh/hr) between OD pair w. Consider link flow is Q_a, flow on route r between OD pair w is f_{rw}, cost of route r between OD pair w is c_{rw}, link capacity is C_a, link length is L_a, link free flow travel time is t_a^0 (hr/km), link travel time is t_a (hr/km), link travel cost is T_a, mileage at free flow speed (km/lt) is M_a^0, link free flow speed is V_a^0 (kmph), link speed is V_a (kmph), fuel cost is F (price/lt) and value of time is VOT (price/hr). vehicle emission factor is e (Q_a) (gm/km) which depends on speed and speed depends on link flows through speed-flow relationship. Vehicle emission factor using approximation curve is e_a^{approx} (Q_a) (gm/km) and vehicle emission factor using original curve is e_a^{org} (Q_a) (gm/km). Link emission standard level (permissible emissions) is $\bar{E}_a$ (gm/km-hr), total link emissions using approximation curve is $E_a^{approx}(Q_a)$ (gm/km-hr) and total link emissions using original curve is $E_a^{org}(Q_a)$ (gm/km-hr). Davidson time-flow equation is

$$t_a = t_a^0\left[1+\frac{J_D X_a}{(1-X_a)}\right] \quad (1)$$

Where J_D is delay parameter and X_a is link saturation ratio (Q_a/C_a).

3.3 Total link costs

Total link travel cost (price units) is given by

$$T_a = \left[VOT * L_a * t_a^0 + \frac{FL_a}{M_a^0}\right]\left[1+\frac{J_D X_a}{(1-X_a)}\right] \quad (2)$$

In equation (2), the first term is link travel time cost and the second term is link fuel consumption cost. The cost function is flow-dependent and explicitly takes into account the fact that traffic congestion will lead to a higher generalized cost through the opportunity cost of waiting in queues or the payment of a congestion toll. The costs are monotonically increasing with flows and hence it is convex.

3.4 Emission factor and link emissions

The original relationship of speed- NOx emission factor curve for diesel car is given by (Finland curves, FHWA (2005))

$$e = -10^{-7}V^3 + 0.0002\,V^2 - 0.0245\,V + 1.3698 \qquad (3)$$

The equation is third order polynomial and the emission factor will increase at later portion of the curve at higher speeds also (Chen, 2005). If the original relation is considered, then the network link pricing should be fixed first for which the equilibrium flows will be simulated using general UE principle. Emissions are calculated for the resulted flows and constraints will be checked whether or not within permissible limits and pricing will be changed and tried again if necessary. This approach is called as descriptive approach modeling. But for optimization problem, due to difficult in computations and non-convexity of the problem, the vehicle average speed and emission factor relationship is approximated as

$$e=2.7331V^{-0.3692} \qquad (4)$$

The emission factor decreases monotonically with average speed and hence it is convex. This assumption may result lower tolls than actual at speeds above 70kmph due to reduced emissions at higher flows which is not true in practice. Figure 1 shows the speed-emission factor relationship. Figure 2 shows the shape of the link flow-link emissions relationship. Total Link emissions are the product of vehicle emission factor and link flows.

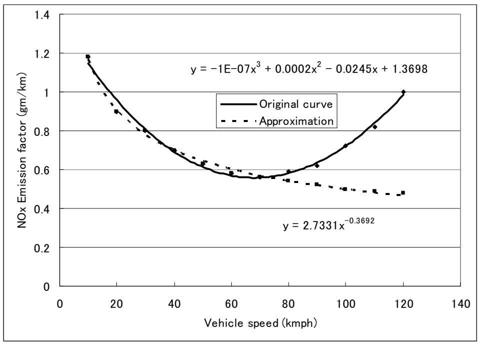

Figure 1: Speed-emission factor relationship

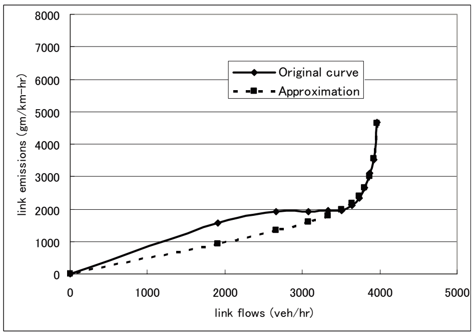

Figure 2: Shape of link flow-link emissions relationship

Approximation emission factor

$$e_a^{approx} = 2.7331\left[V_a^0 / \{1+\left[J_D X_a / (1-X_a)\right]\}\right]^{-0.3692}$$

$$= 2.7331\left[\frac{\{(C_a - Q_a)V_a^0\}}{\{C_a - Q_a + J_D Q_a\}}\right]^{-0.3692} \qquad (5)$$

Total emissions for approximation curve

$$E_a^{approx}(Q_a) = 2.7331\,Q_a\left[\frac{\{(C_a - Q_a)V_a^0\}}{\{C_a - Q_a + J_D Q_a\}}\right]^{-0.3692} \qquad (6)$$

Original emission factor

$$e_a^{org} = -10^{-7}\left[\frac{\{(C_a - Q_a)V_a^0\}}{\{C_a - Q_a + J_D Q_a\}}\right]^3 + 0.0002\left[\frac{\{(C_a - Q_a)V_a^0\}}{\{C_a - Q_a + J_D Q_a\}}\right]^2 - 0.0245\left[\frac{\{(C_a - Q_a)V_a^0\}}{\{C_a - Q_a + J_D Q_a\}}\right] + 1.368 \qquad (7)$$

Total emissions for original curve

$$E_a^{org}(Q_a) = -10^{-7}Q_a\left[\frac{\{(C_a - Q_a)V_a^0\}}{\{C_a - Q_a + J_D Q_a\}}\right]^3 + 0.0002Q_a\left[\frac{\{(C_a - Q_a)V_a^0\}}{\{C_a - Q_a + J_D Q_a\}}\right]^2 - 0.0245Q_a\left[\frac{\{(C_a - Q_a)V_a^0\}}{\{C_a - Q_a + J_D Q_a\}}\right] + 1.368Q_a \qquad (8)$$

For any given link emission standards ($\bar{E}_a$) for the optimum approach, the link environmental capacity ($Q_a^{\bar{E}}$) can be determined by solving eq.(6), where $E_a^{approx}(Q_a)= \bar{E}_a$ and $Q_a=Q_a^{\bar{E}}$. For any positive flows and emission standards, environmental capacity is below the physical capacity, $Q_a^{\bar{E}} < C_a$. Here the approximation curve is used for optimum solution for which augmented Lagrange algorithm is used.

3.5 Model formulation

The objective is to model optimum Environmental Road Pricing considering local emissions of road transportation network. The model should keep the local emissions below permissible emissions on selected locations based on the sensitivity of the location. General congestion pricing or UE models may not always keeps the local emissions below the permissible emissions on all selected locations due to the optimization of costs alone. Similarly, total minimization of emissions may not always keep the individually selected location emissions below the permissible emissions. Hence, it is necessary to model Environmental Road Pricing considering local emission constraints as main objective.

The existing congestion pricing or UE model concepts can be used for optimum Environmental Road Pricing by using environmental costs implicitly with the consideration of emission constraints in the model. The optimum Environmental Road Pricing combined with Congestion Pricing (CP+ERP) can be formulated by extending general congestion pricing models with the inclusion of emission constraints. The optimum Environmental Road Pricing (ERP) can be formulated from general UE model by including selected link emission constraints.

In CP+ERP and ERP options, the optimum tolls are determined by implicitly considering the environmental costs in terms of link emission constraints. These link emission constraints are transformed in terms of link environmental capacity constraints for all selected locations. As the system optimization concept is well received by analysts and policy makers, here the model formulation is shown for Environmental Road Pricing combined with Congestion Pricing (CP+ERP) and ERP option is shown in the end. Both the options will keep the emissions below the permissible levels.

Model formulation to minimize the total system cost by keeping local emissions below permissible emissions on selected locations is given by

$$\min Z(Q) = \sum_{a\in A} Q_a T_a(Q_a) \tag{9}$$

Subjected to $Q_a \le Q_a^{\bar{E}}$, $a\in A$ (10)

$$\sum_{r\in R_w} f_{rw} = d_w, w \in W \tag{11}$$

$f_{rw} \ge 0$, ,$r\in R_w$,$w\in W$ (12)

Eq.(9) minimizes system cost, Eq.(10) restricts the link flows below link environmental capacity and the optimal tolls are determined by making Lagrangean formation with dual variables λ_a, for the constraints, Eq.(11) checks the sum of all route flows should be equal to OD demand and Eq.(12) checks the route flows should be non-negative.

$\Omega=\{Q \mid Q =\Delta f, \Lambda f=d, f\ge 0\}$ (13)

where link/path incidence matrix $\Delta=[\delta_{ar}]$ and the OD pair/path incidence matrix $\Lambda= [\Lambda_{rw}]$. $\Lambda_{rw}=1$ if $r\in R_w$ and 0 otherwise. Path flow vector f is defined as $f^*=(f_{rw}^*, r\in R_w, w\in W)^T$ and column vector of all OD demands d is defined as $d=(d_w, w\in W)^T$.

Link flows, $Q=(Q_a, a\in A)^T$, are defined by

$$Q_a = \sum_{w\in W} \sum_{r\in R_W} f_{rw}\delta_{ar}, a \in A \tag{14}$$

View link flow Q as a function of path flow vector f as defined by link-path flow relation and construct Lagrangean function

$$\min L(f,\mu,\lambda) = \sum_{a\in A} Q_a T_a(Q_a) + \sum_{a\in A} \lambda_a\left(Q_a - Q_a^{\bar{E}}\right) + \sum_{w\in W} \mu_w \left[d_w - \sum_{r\in R_W} f_{rw}\right] \tag{15}$$

The first order optimality conditions are

$$f_{rw}\frac{\partial L(f^*,\mu,\lambda)}{\partial f_{rw}} = f_{rw}^*(c_{rw} - \mu_w) = 0, r \in R_w, w \in W \tag{16}$$

$$\frac{\partial L(f^*,\mu,\lambda)}{\partial f_{rw}} = c_{rw} - \mu_w \ge 0, r \in R_w, w \in W \tag{17}$$

$$\frac{\partial L(f^*,\mu,\lambda)}{\partial \mu_w} = \sum_{r\in R_W} f_{rw}^* - d_w = 0, w \in W \tag{18}$$

$$\frac{\partial L(f^*,\mu,\lambda)}{\partial \lambda_a} = Q_a - Q_a^{\bar{E}} \le 0 \Rightarrow Q_a \le Q_a^{\bar{E}}, a \in A \tag{19}$$

$$\lambda_a\left(Q_a - Q_a^{\bar{E}}\right) = 0, a \in A \tag{20}$$

$$\lambda_a \ge 0, a \in A \tag{21}$$

Where λ_a, $a\in A$ and μ_w are the Lagrange multipliers associated with constraints (10) & (11). μ_w is the minimum travel cost between OD pair $w\in W$ associated with constraint (11) and $\mu=(\mu_w, w\in W)^T$. From optimality conditions (16)–(21), for any optimum solution, the following relations can be obtained.

$c_{rw}=\mu_w$ if $f_{rw}^*>0$ and $c_{rw}\ge\mu_w$ if $f_{rw}^*=0$, $r\in R_w$, $w\in W$ (22)

$\lambda_a=0$, if $Q_a^*<Q_a^{\bar{E}}$ and $\lambda_a\ge 0$, if $Q_a^*=Q_a^{\bar{E}}$, $a\in A$ (23)

Route costs c_{rw} are given by

$$c_{rw} = \sum_{a\in A}\left(T_a(Q_a) + Q_a dT_a(Q_a)/dQ_a + \lambda_a\right)\delta_{ar} \tag{24}$$

$$c_{rw} = \sum_{a\in A} \overline{T}_a(Q_a)\delta_{ar}, r \in R_w, w \in W \tag{25}$$

Link costs are given by

$$\overline{T}_a(Q_a) = T_a(Q_a) + Q_a dT_a(Q_a)/dQ_a + \lambda_a \tag{26}$$

Emissions exceed only ($\lambda_a>0$) when flows reached environmental capacity ($Q_a^*=Q_a^{\bar{E}}$). Below environmental capacity, link costs will be solely given by

$T_a(Q_a)+Q_a dT_a(Q_a)/d(Q_a)$.

The marginal-cost toll evaluated at traffic flow Q_a, $a\in A$, represents $Q_a\ dT_a\ (Q_a)/d\ Q_a + \lambda_a$. The excess emissions, λ_a, $a\in A$ are above the standard levels and should be substituted by an additional toll. Consequently, the optimal link toll to be charged is given by

$$u_a = Q_a^{*EC}\ dT_a\ (Q_a^{*EC})/d\ Q_a^{*EC} \quad +\lambda_a,\ a\in A \qquad (27)$$

Where Q_a^{*EC}, $a\in A$ denotes the optimum solution to the system optimum problem with fixed demand and environment constraints.

$$\lambda_a=0,\ \text{if } Q_a^{*EC}<Q_a^{\bar{E}}\ \text{ and } \lambda_a\geq 0,\ \text{if } Q_a^{*EC}=Q_a^{\bar{E}},\ a\in A \qquad (28)$$

The optimum link toll is sum of congestion pricing and environmental road pricing. Hence, the formulation (9)-(12) can be called as combined congestion and environmental road pricing (CP+ERP). In eq.(28), the environmental road pricing will be zero, if the link flows are below the environmental capacity.

Other pricing options such as Congestion Pricing (CP) and Environmental Road Pricing (ERP) can also be formulated with slight changes in the formulation (9)-(12). In the formulation, if the emission constraints (eq.(10)) are neglected, then it will be congestion pricing model and the optimum tolls are just congestion pricing(CP). Link costs for CP option are given by

$$\overline{T}_a(Q_a)=\widetilde{T}_a(Q_a)=T_a(Q_a)+Q_a dT_a(Q_a)/dQ_a \qquad (29)$$

In eq.(29), link costs are the sum of average cost and congestion toll (external delay costs). In the formulation (9)-(12), if the congestion delay costs are neglected in eq.(9), then it will be Environmental Road Pricing model and the optimum tolls are just Environmental Road Pricing (ERP). For ERP option, eq.(9) has to be replaced in the formulation with the equation

$$\min Z(Q)=\sum_{a\in A}\int_0^{Q_a} T_a(\omega)d\omega \qquad (30)$$

The link costs for ERP option are given by

$$\overline{T}_a(Q_a)=T_a(Q_a)+\lambda_a \qquad (31)$$

In eq. (31), link costs are the sum of average cost and environmental road pricing. These are similar to the link costs of UE problem with link emission constraints.

CP+ERP option and ERP options will always keeps the local emissions below permissible emissions for all selected locations due to the inclusion emission constraints. CP option may not always keeps the local emissions below the permissible emissions for all selected locations. Hence either of CP+ERP option and ERP option can be used to keep the local emissions below the permissible emissions on all selected locations. ERP option is considered in this paper for validation purpose of the accuracy of optimum tolls by augmented Lagrange algorithm.

4. Solution algorithm

The approximation curve is used for optimum approach solution. Many algorithms employ a strategy that converts constraint problem into a un- constraint problems through a penalized/dualization of the constraints, so that various existing efficient approaches can be applied for the solution. Augmented Lagrangian algorithm is one of the efficient and locally convergent methods for the optimization problem with nonlinear constraints (Bertsekas, 1982, Yang Hai et.al, 2005). The discontinuity of the second derivatives of the sub problem of Augmented Lagrangian algorithm is not a serious inconvenient in practical computations and was the best one among the various methods (Birgin et.al, 2005). Augmented Lagrangian algorithm functional form with Z and emission constraint (10) is given by

$$L_\rho(Q,\lambda)=Z(Q) + \sum_{a\in A}\frac{1}{2\rho}\left[\max{}^2\left\{0,\lambda_a+\rho\left(Q_a-Q_a^{\bar{E}}\right)\right\}-\lambda_a^2\right] \qquad (32)$$

where λ is the vector of dual variables associated with emission constraints. Augmented Lagrangean dual algorithm can be summarized as below.

Step0: (Initialization) Initialize the feasible set of link flows $Q_a^{(0)}$. Select the initial values λ_a, for all links, $a\in A$.

$$\lambda_0^{(0)}=T_a(Q_a^{(0)})-T_a(Q_a^{\bar{E}})\text{if } Q_a^{(0)}>Q_a^{\bar{E}} \text{ and 0 otherwise} \qquad (33)$$

Initial flows can be selected in between zero and physical capacity. Choose initial $\rho>0$ and set the iteration counter n:=0. Initial ρ can be taken as 0.01.

Step1: Solve the traffic assignment problem with link costs $T_a(Q_a)+\max\{0, \lambda_a^{(n)}+\rho^{(n)}(Q_a-Q_a^{\bar{E}})\}$, $a\in A$ for ERP option to get link flow pattern $Q_a^{(n)}$. Frank-wolf algorithm can be used for traffic assignment in each step. The traffic assignment can be terminated when the error

$$\varepsilon=\left[\sqrt{\sum_{a\in A}\left(Q_a^{(p+1)}-Q_a^{(p)}\right)^2}\right]\Big/\sum_{a\in A}Q_a^{(p)}<0.000001 \qquad (34)$$

Here p is the iterative steps in assignment.

Step2: (Termination) if $\max\limits_{a\in A}\left[\left|Q_a^{(n)} - Q_a^{\bar{E}}\right|\lambda_a^{(n)}\right] < 0.0001$,

Stop and accept $Q^{(n)}$, $\lambda^{(n)}$ as the solution. Otherwise update the multiplier and penalty parameter as in step3. Termination criteria check the slackness condition in terms of link flows and multiplier. The termination criteria will be satisfied if $\lambda^{(n)}$ is zero when $Q_a<Q_a^{\bar{E}}$ because of its multiplication ($|Q_a-Q_a^{\bar{E}}|\lambda^{(n)}$) is zero. The termination criteria may also be satisfied even if $\lambda^{(n)}\geq 0$ when $|Q_a-Q_a^{\bar{E}}|$ is very small value or zero. Tolerance may vary as per desired degree of accuracy.

Step3: Lagrange multipliers can be updated based on the assigned flows ($Q^{(n)}$) in the previous iteration. The updated multiplier $\lambda_a^{(n+1)}$ for the next iteration is given by

$\lambda_a^{(n+1)} = \max\{0, \lambda_a^{(n)} + \rho^{(n)}(Q^{(n)}(\rho, \lambda^{(n)}) - Q_a^{\bar{E}})\}, a\in A$ (35)

The idea of updating of penalty parameter is that the penalty parameter ρ will not be changed if the last inner iteration produced a significative improvement both of feasibility and complementarity. For updating of ρ value for n+1 step, requires the comparison of solutions in n & n-1 steps. If n=0 then $\rho^{(n+1)} = \rho^{(1)} = 0.01$(same as initial value), otherwise

$$\rho^{(n+1)} = \begin{cases} \kappa\rho^{(n)}, \text{if } \sum_{a\in A}\max^2\left\{-\lambda_a^{(n)}/\rho^{(n)}, \left(Q_a^{(n)} - Q_a^{\bar{E}}\right)\right\} \\ \qquad > \gamma^2 \sum_{a\in A}\max^2\left\{-\lambda_a^{(n-1)}/\rho^{(n-1)}, \left(Q_a^{(n-1)} - Q_a^{\bar{E}}\right)\right\} \\ \rho^{(n)}, \text{otherwise} \end{cases} \tag{36}$$

where $2\leq\kappa\leq 10$ and $\gamma=0.25$ can be chosen. Flows $Q^{(n)}$ can be used as initial flows for the next step. Set n:=n+1 and go to Step 1. Min $L_\rho(Q,\lambda)$ subject to $Q\in\Omega_Q$ is strictly convex, so all existing efficient algorithms for solving standard traffic assignment problems can be used in Step 1. Step 2 carries out a safe-guard termination based on infeasibility of environmental capacity (emission standard levels).

5. Hypothetical data analysis for Validation

Hypothetical data analysis aimed to illustrate the model and validate the pricing of optimum solution with descriptive approach using general UE model. Consider the hypothetical network as shown in Figure 3 with two OD pairs (AC & BC) with equal demands. A common link (5) also chosen to see the effect of link interactions on optimum tolls. Assume delay parameter J_D= 0.1, fuel cost = 1 price/lt, Value of Time= 20 price/hr, Mileage is 35km/lt at free flow speed 120 kmph. Consider link capacities (veh/hr) C_1=4000, C_2=3000, C_3=3000, C_4=4000, C_5=3500 and link lengths L_1=5km, L_2=4km, L_3=4km, L_4=4km, L_5=3km respectively for link1, link2, link3, link4 and link5. The network routes are $R_1=L_1$, $R_2=L_2+L_5$ for OD pair AC and $R_3=L_3+L_5$, $R_4=L_4$ for OD pair BC; Tolls are in price units.

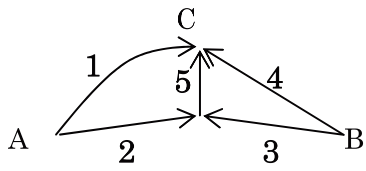

Figure 3: Hypothetical Network

The road pricings of optimum approach using approximation curve by augmented Lagrange algorithm are compared with the optimum pricing of descriptive approach using approximation curve with standard UE model under the same conditions for its validity. ERP option is considered for validation purpose of the accuracy of optimum tolls by augmented Lagrange algorithm. The optimum tolls are obtained by the augmented lagrangean algorithm. The optimum tolls of descriptive approach are obtained by choosing the solution which is lowest system cost that satisfies the emissions constraints on all selected links in the combinatorial approach of varying link pricings and their corresponding link emissions. A suitable link price range of 0 to 1 is considered with a stepwise increment of 0.05 in the illustration. So the actual optimum may differ in fractions less than 0.05 and the optimum approach solution differs upto 0.05 with descriptive approach solution may be acceptable. Table 1 shows the comparison of road pricings of both approaches for the above data. By comparing both the approaches link pricings, it can be observed that the difference is negligible and is due to the minimum stepwise increment of 0.05 considered in UE model. For example, in case of two OD pairs (AC & BC) with equal demands of 3250veh/hr (total demand 6500 veh/hr) each and emission standards (gm/km-hr) for link1 and link4 are $\bar{E}_1$ = 2000, $\bar{E}_4$ = 1500, the optimum link tolls are 0, 0.385 and link flows are 3212, 2937 respectively for link1 and link4. The system cost is 8957 price units. The optimum pricings of descriptive approach with general UE model are 0, 0.40 and the flows are 3208, 2893 respectively for link1 and link4.The system cost is 8960 price units. By comparing both the pricing, the difference found is 0.015 and is negligible. It was also observed that by keeping the same optimum pricing (0, 0.385) as input in UE model, the same solution (0, 0.385) was found to be the optimist solution compared to all other in the set of solutions. So the optimum approach estimates the accurate optimum solution with augmented Lagrangean algorithm and can be used in general networks.

Table 1: Comparison of road pricing of optimum and descriptive approaches

Demand (veh/hr)	$\bar{E}_1$=2000,$\bar{E}_4$=1000		$\bar{E}_1$=2000,$\bar{E}_4$=1500	
	Optimum	UE	Optimum	UE
5500	p_1=0	p_1=0	p_1=0	p_1=0
	p_4=0.540	p_4=0.55	p_4=0	p_4=0
6500	p_1=0	p_1=0	p_1=0	p_1=0
	p_4=0.584	p_4=0.60	p_4=0.385	p_4=0.40
7500	p_1=0	p_1=0	p_1=0	p_1=0
	p_4=0.690	p_4=0.70	p_4=0.433	p_4=0.45

Optimum -Optimum approach using approximation curve

UE - Descriptive approach using approximation curve with UE model

p_1=pricing on link1, p_4=pricing on link4

6. Conclusions

The model for optimum tolls to keep the local emissions below the permissible levels in selected locations are proposed and validated with hypothetical network. The difference in optimum pricing of both approaches using augmented Lagrangean algorithm and general UE model are negligible and is due to the step size increments of 0.05 considered in descriptive approach with UE model. Therefore, the augmented Lagrange algorithm estimates the accurate and exact optimum solution. The model can be used for optimum tolls and keeps the emissions below the permissible levels for all selected locations.

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Appendix

Congestion Pricing (CP) formulation given in the literature (Yang et.al, 2005 and others) by

$$\min Z(Q) = \sum_{a \in A} Q_a T_a(Q_a) \qquad (A)$$

$$\sum_{r \in R_w} f_{rw} = d_w, \; w \in W \qquad (B)$$

$$f_{rw} \ge 0, \; ,r \in R_w, w \in W \qquad (C)$$

Link costs for CP option are given by

$$\overline{T}_a(Q_a) = \widetilde{T}_a(Q_a) = T_a(Q_a) + Q_a dT_a(Q_a)/dQ_a$$

Environmental Road Pricing (ERP) is given by

$$\min Z(Q) = \sum_{a \in A} \int_0^{Q_a} T_a(\omega) d\omega \qquad (D)$$

Subjected to $Q_a \le Q_a^{\bar{E}}$, $a \in A$ (E)

$$\sum_{r \in R_w} f_{rw} = d_w, \; w \in W \qquad (F)$$

$$f_{rw} \ge 0, \; ,r \in R_w, w \in W \qquad (G)$$

Link costs for ERP option are given by

$$\overline{T}_a(Q_a) = T_a(Q_a) + \lambda_a$$

Combined Congestion Pricing and Environmental Road Pricing (CP+ERP) is given by

$$\min Z(Q) = \sum_{a \in A} Q_a T_a(Q_a) \qquad (H)$$

Subjected to $Q_a \le Q_a^{\bar{E}}$, $a \in A$ (I)

$$\sum_{r \in R_w} f_{rw} = d_w, \; w \in W \qquad (J)$$

$$f_{rw} \ge 0, \; ,r \in R_w, w \in W \qquad (K)$$

Link costs for CP+ERP option are given by

$$\overline{T}_a(Q_a) = T_a(Q_a) + Q_a dT_a(Q_a)/dQ_a + \lambda_a$$

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